

Paper Reference 9FM0/02
Pearson Edexcel
Level 3 GCE

Further Mathematics
Advanced
PAPER 2: Core Pure Mathematics 2

Monday 3 June 2024 – Afternoon

Time: 1 hour 30 minutes

YOU MUST HAVE
Mathematical Formulae and Statistical Tables (Green),
calculator

YOU WILL BE GIVEN
Diagram Booklet
Answer Booklet

X75683A

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

INSTRUCTIONS

In the boxes on the Answer Booklet and on the Diagram Booklet, write your name, centre number and candidate number.

Answer ALL questions and ensure that your answers to parts of questions are clearly labelled.

Answer the questions in the Answer Booklet – there may be more space than you need.

Do NOT write on this Question Paper.

You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

Inexact answers should be given to three significant figures unless otherwise stated.

INFORMATION

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

There are **9** questions in this Question Paper.
The total mark for this paper is **75**.

The marks for **EACH** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

ADVICE

Read each question carefully before you start to answer it.

Try to answer every question.

Check your answers if you have time at the end.

1. (a) Using the definition of $\sinh x$ in terms of exponentials, prove that

$$4 \sinh^3 x + 3 \sinh x \equiv \sinh 3x$$

(2 marks)

- (b) Hence solve the equation

$$\sinh 3x = 19 \sinh x$$

giving your answers as simplified natural logarithms where appropriate.

(5 marks)

(Total for Question 1 is 7 marks)

2. $f(x) = \tanh^{-1} \left(\frac{3-x}{6+x} \right) \quad |x| < \frac{3}{2}$

(a) Show that

$$f'(x) = -\frac{1}{2x+3}$$

(4 marks)

(b) Hence determine $f''(x)$

(1 mark)

(c) Hence show that the Maclaurin series for $f(x)$, up to and including the term in x^2 , is

$$\ln p + qx + rx^2$$

where p , q and r are constants to be determined.

(3 marks)

(Total for Question 2 is 8 marks)

3. (a) Explain why

$$\int_{\frac{4}{3}}^{\infty} \frac{1}{9x^2 + 16} dx$$

is an improper integral.

(1 mark)

(b) Show that

$$\int_{\frac{4}{3}}^{\infty} \frac{1}{9x^2 + 16} dx = k\pi$$

where k is a constant to be determined.

(4 marks)

(Total for Question 3 is 5 marks)

4. Use the method of differences to show that

$$\sum_{r=1}^n \frac{2}{(r+4)(r+6)} = \frac{n(an+b)}{30(n+5)(n+6)}$$

where **a** and **b** are integers to be determined.

(6 marks)

(Total for Question 4 is 6 marks)

5. The locus **C** is given by

$$|z - 4| = 4$$

The locus **D** is given by

$$\arg z = \frac{\pi}{3}$$

(a) Sketch, on the same Argand diagram, the locus **C** and the locus **D**

There are blank axes on pages 34 – 45 in the Answer Booklet if you wish to use them.

(4 marks)

(continued on the next page)

5. continued.

The set of points A is defined by

$$A = \left\{ z \in \mathbb{C} : |z - 4| \leq 4 \right\} \cap \left\{ z \in \mathbb{C} : 0 \leq \arg z \leq \frac{\pi}{3} \right\}$$

(b) Show, by shading on your Argand diagram, the set of points A

(1 mark)

(c) Find the area of the region defined by A , giving your answer in the form

$$p\pi + q\sqrt{3}$$

where p and q are constants to be determined.

(4 marks)

(Total for Question 5 is 9 marks)

6. The motion of a particle **P** along the **x**–axis is modelled by the differential equation

$$2\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 2x = 4t + 12$$

where **P** is **x** metres from the origin **O** at time **t** seconds, $t \geq 0$

- (a) Determine the general solution of the differential equation.

(6 marks)

- (b) Hence determine the particular solution for which

$$x = 3 \text{ and } \frac{dx}{dt} = -2 \text{ when } t = 0$$

(3 marks)

(continued on the next page)

6. continued.

(c) (i) Show that, according to the model, the minimum distance between **O and **P** is $(2 + \ln 2)$ metres.**

(ii) Justify that this distance is a minimum.
(4 marks)

For large values of t the particle is expected to move with constant speed.

(d) Comment on the suitability of the model in light of this information.
(1 mark)

(Total for Question 6 is 14 marks)

7. (a) Determine the roots of the equation

$$z^6 = 1$$

giving your answers in the form

$$e^{i\theta} \text{ where } 0 \leq \theta < 2\pi$$

(2 marks)

- (b) Show the roots of the equation in part (a) on a single Argand diagram.

There are blank axes on pages 34 – 45 in the Answer Booklet if you wish to use them.

(2 marks)

- (c) Show that

$$\left(\sqrt{3} + i\right)^6 = -64$$

(2 marks)

(continued on the next page)

7. continued.

(d) Hence, or otherwise, solve the equation

$$z^6 + 64 = 0$$

giving your answers in the form

$re^{i\theta}$ where $0 \leq \theta < 2\pi$

(3 marks)

(Total for Question 7 is 9 marks)

8. $A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ k & 3 & 6 \end{pmatrix} \quad k \neq 0$

(a) Find, in terms of k , A^{-1}
(4 marks)

(b) Determine, in simplest form in terms of k , the coordinates of the point where the following planes intersect.

$$3x + y - z = 3$$

$$x + y + z = 1$$

$$kx + 3y + 6z = 6$$

(3 marks)

(Total for Question 8 is 7 marks)

9. Refer to Diagram 1 and Diagram 2 for Question 9 in the Diagram Booklet.

Diagram 1 shows the central vertical cross-section **ABCDEFA** of a vase together with measurements that have been taken from the vase.

The horizontal cross-section between **AB** and **FC** is a circle with diameter **4 cm**

The base of the vase **ED** is horizontal and the point **E** is vertically below **F** and the point **D** is vertically below **C**

(continued on the next page)

9. continued.

Using these measurements, the curve **CD** is modelled by the parametric equations

$$x = a + 3 \sin 2t$$

$$y = b \cos t$$

$$0 \leq t \leq \frac{\pi}{2}$$

where **a** and **b** are constants and **O** is the fixed origin, as shown in Diagram 2.

(a) Determine the value of **a** and the value of **b** according to the model.

(2 marks)

(b) Using algebraic integration and showing all your working, determine, according to the model, the volume of the vase, giving your answer to the nearest cm^3

(7 marks)

(continued on the next page)

Turn over

9. continued.

(c) State a limitation of the model.

(1 mark)

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END OF PAPER
